PROPERTIES OF DETERMINANTS

Created: Monday, 28 November 2011 09:38 | Published: Monday, 28 November 2011 09:38 | Written by Super User | Print

Property I

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \overbrace{a_{11}}^{\text{First row element}} \overbrace{\left(a_{22}a_{33} - a_{23}a_{32}\right)}^{\text{Cofactor}} \overbrace{-}^{\text{Alternating signs}} \overbrace{a_{12}}^{\text{First row element}} \overbrace{\left(a_{21}a_{33} - a_{23}a_{31}\right)}^{\text{Cofactor}}$$
$$+ a_{13}\left(a_{21}a_{32} - a_{22}a_{31}\right)$$

The value of the determinant remains unchanged if its rows and columns are interchanged.

For example: $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 0 & 1 \\ -1 & 4 & 0 \end{vmatrix}$

= 1 (0 - 4) – 2 (0 – (-1)) + 3 (8 - 0)

= -4 -2 + 24 = 18 … (i)

Let $\Delta' = \begin{vmatrix} 1 & 2 & -1 \\ 2 & 0 & 4 \\ 3 & 1 & 0 \end{vmatrix}$

= 1 (0 - 4) - 2 (0 - 12) – 1 (2 - 0)

= -4 + 24 – 2 = 18

Hence, ? = ?’

Property II

If any two rows (or columns) of a determinant are interchanged, then sign of determinant changes.

For example: We know ? = 18 from [equation (i) above]

Let us interchange 2nd and 3rd rows of ?’ and find its value

$\Delta' = \begin{vmatrix} 1 & 2 & -1 \\ 3 & 1 & 0 \\ 2 & 0 & 4 \end{vmatrix}$

= 1 (4 - 0) - 2 (12 - 0) – 1 (0 - 2)

= 4 – 24 + 2 = -18

Hence, ?’ = -?

Property III

If any two rows (or columns) of a determinant are identical, then the value of the determinant is zero.

For example: $\Delta = \begin{vmatrix} 2 & 0 & -1 \\ 1 & 1 & 4 \\ 2 & 0 & -1 \end{vmatrix} \quad [R_1 = R_3]$

= 2[-1 - 0] - 0[-1 - 8] - 1[0 - 2]
= -2 – 0 + 2 = 0

Property IV

If each element of a row (or a column) of a determinant is multiplied by a constant ‘k’, then its value gets multiplied by k

$$\text{Let } \Delta = \begin{vmatrix} ka_1 & kb_1 & kc_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = k \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Property V

If some or all elements of a row or column of a determinant are expressed as sum of two (or more) terms, then the determinant can be expressed as sum of two (or more) determinants.

$$\begin{vmatrix} a_1+\lambda_1 & a_2+\lambda_2 & a_3+\lambda_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} + \begin{vmatrix} \lambda_1 & \lambda_2 & \lambda_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

Property VI

The value of the determinant remains same if we apply the operation $R_i \longrightarrow R_i + kR_j$ or $C_i \longrightarrow C_i + kC_j$
For example: Using[properties](#) of determinants: Solve

$$\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix} = 0$$

1. $\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix} = \begin{vmatrix} 1 & bc & ab+ac \\ 1 & ca & bc+ab \\ 1 & ab & ac+bc \end{vmatrix}$

$$= \begin{vmatrix} 1 & bc & ab+ac+bc \\ 1 & ca & bc+ab+ca \\ 1 & ab & ac+bc+ab \end{vmatrix} \quad C_3 \longrightarrow C_2 + C_3$$

$= ab+ac+bc \begin{vmatrix} 1 & bc & 1 \end{vmatrix}$

= (ab + ac + bc) x 0

= 0

Now try it yourself! Should you still need any help, click here to schedule live online session with e Tutor!

About eAge Tutoring:

eAgeTutor.com is the premium online tutoring provider. Using materials developed by highly qualified educators and leading content developers, a team of top-notch software experts, and a group of passionate educators, eAgeTutor works to ensure the success and satisfaction of all of its students.

Contact us today to learn more about our tutoring programs and discuss how we can help make the dreams of the student in your life come true!

Reference Links:

- http://en.wikipedia.org/wiki/Determinant
- http://en.wikipedia.org/wiki/Row_and_column_spaces
- http://en.wikipedia.org/wiki/Determinant#Properties_characterizing_the_determinant

Category:ROOT