## PROPERTIES OF DETERMINANTS

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## Property I

$$
\left|\begin{array}{lll}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{array}\right|=a_{11}\left(a_{22} a_{33}-a_{23} a_{32}\right)-a_{12}\left(a_{21} a_{33}-a_{23} a_{31}\right)
$$

The value of the determinant remains unchanged if its rows and columns are interchanged.

For example: $\Delta=\left|\begin{array}{ccc}1 & 2 & 3 \\ 2 & 0 & 1 \\ -1 & 4 & 0\end{array}\right|$

$$
\begin{aligned}
& \quad=1(0-4)-2(0-(-1))+3(8-0) \\
& \quad \text { Let } \quad \begin{array}{lll}
\Delta^{\prime} & =\left|\begin{array}{lll}
1 & 2 & -1 \\
2 & 0 & 4 \\
3 & 1 & 0
\end{array}\right| \\
= & 1(0-4)-2(0-12)-1(2-0) \\
= & -4+24-2=18
\end{array}
\end{aligned}
$$

Hence, ? = ?

## Property II

If any two rows (or columns) of a determinant are interchanged, then sign of determinant changes.
For example: We know ? = 18 from [equation (i) above]
Let us interchange 2 nd and 3rd rows of ?' and find its value

$$
\begin{aligned}
\Delta^{\prime} & =\left|\begin{array}{ccc}
1 & 2 & -1 \\
3 & 1 & 0 \\
2 & 0 & 4
\end{array}\right| \\
& =1(4-0)-2(12-0)-1(0-2) \\
& =4-24+2=-18
\end{aligned}
$$

Hence, ? $=-$ ?

## Property III

If any two rows (or columns) of a determinant are identical, then the value of the determinant is zero.

For example:

$$
\begin{aligned}
& \Delta=\left|\begin{array}{ccc}
2 & 0 & -1 \\
1 & 1 & 4 \\
2 & 0 & -1
\end{array}\right|\left[R_{1}=R_{3}\right] \\
& =2[-1-0]-0[-1-8]-1[0-2] \\
& =-2-0+2=0
\end{aligned}
$$

## Property IV

If each element of a row (or a column) of a determinant is multiplied by a constant ' $k$ ', then its value gets multiplied by $k$ Let $\Delta=\left|\begin{array}{lll}k a_{1} & k b_{1} & k c_{1} \\ a_{2} & b_{2} & c_{2} \\ a_{3} & b_{3} & c_{3}\end{array}\right|=k\left|\begin{array}{lll}a_{1} & b_{1} & c_{1} \\ a_{2} & b_{2} & c_{2} \\ a_{3} & b_{3} & c_{3}\end{array}\right|$

## Property V

If some or all elements of a row or column of a determinant are expressed as sum of two (or more) terms, then the determinant can be expressed as sum of two (or more) determinants.
$\left|\begin{array}{ccc}a_{1}+\lambda_{1} & a_{2}+\lambda_{2} & a_{3}+\lambda_{3} \\ b_{1} & b_{2} & b_{3} \\ c_{1} & c_{2} & c_{3}\end{array}\right|=\left|\begin{array}{lll}a_{1} & a_{2} & a_{3} \\ b_{1} & b_{2} & b_{3} \\ c_{1} & c_{2} & c_{3}\end{array}\right|+\left|\begin{array}{ccc}\lambda_{1} & \lambda_{2} & \lambda_{3} \\ b_{1} & b_{2} & b_{3} \\ c_{1} & c_{2} & c_{3}\end{array}\right|$

## Property VI

The value of the determinant remains same if we apply the operation $R_{i} \longrightarrow R_{i}+k R_{j}$ or $C_{i} \longrightarrow C_{i}+k C_{j}$
For example: Usingproperties of determinants: Solve

$$
\left|\begin{array}{lll}
1 & b c & a(b+c) \\
1 & c a & b(c+a) \\
1 & a b & c(a+b)
\end{array}\right|=0
$$

1. $\left|\begin{array}{ccc}1 & b c & a(b+c) \\ 1 & c a & b(c+a) \\ 1 & a b & c(a+b)\end{array}\right|=\left|\begin{array}{lll}1 & b c & a b+a c \\ 1 & c a & b c+a b \\ 1 & a b & a c+b c\end{array}\right|$
$=\left|\begin{array}{lll}1 & b c & a b+a c+b c \\ 1 & c a & b c+a b+c a \\ 1 & a b & a c+b c+a b\end{array}\right| \mathrm{C}_{3} \longrightarrow C_{2}+C_{3}$

$$
\begin{aligned}
& =(a b+a c+b c) \times 0 \\
& =0
\end{aligned}
$$

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## Reference Links:

- http://en.wikipedia.org/wiki/Determinant
- http://en.wikipedia.org/wiki/Row_and_column_spaces
- http://en.wikipedia.org/wiki/Determinant\#Properties_characterizing_the_determinant


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