PROPERTIES OF DETERMINANTS

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Property I

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \overbrace{a_{11}}^{\text{First row element}} \overbrace{\left(a_{22}a_{33} - a_{23}a_{32}\right)}^{\text{Cofactor}} \overbrace{-}^{\text{Alternating signs}} \overbrace{a_{12}}^{\text{First row element}} \overbrace{\left(a_{21}a_{33} - a_{23}a_{31}\right)}^{\text{Cofactor}}$$
$$+ a_{13}\left(a_{21}a_{32} - a_{22}a_{31}\right)$$

The value of the determinant remains unchanged if its rows and columns are interchanged.

For example: $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 0 & 1 \\ -1 & 4 & 0 \end{vmatrix}$

= 1 (0 - 4) – 2 (0 – (-1)) + 3 (8 - 0)

= -4 -2 + 24 = 18 … (i)

Let $\Delta' = \begin{vmatrix} 1 & 2 & -1 \\ 2 & 0 & 4 \\ 3 & 1 & 0 \end{vmatrix}$

= 1 (0 - 4) - 2 (0 - 12) – 1 (2 - 0)

= -4 + 24 – 2 = 18

Hence, ? = ?’

Property II

If any two rows (or columns) of a determinant are interchanged, then sign of determinant changes.

For example: We know ? = 18 from [equation (i) above]

Let us interchange 2nd and 3rd rows of ?’ and find its value

$\Delta' = \begin{vmatrix} 1 & 2 & -1 \\ 3 & 1 & 0 \\ 2 & 0 & 4 \end{vmatrix}$

= 1 (4 - 0) - 2 (12 - 0) – 1 (0 - 2)

= 4 – 24 + 2 = -18

Hence, ?’ = -?

Property III

If any two rows (or columns) of a determinant are identical, then the value of the determinant is zero.

For example: $\Delta = \begin{vmatrix} 2 & 0 & -1 \\ 1 & 1 & 4 \\ 2 & 0 & -1 \end{vmatrix} \quad [R_1 = R_3]$

$= 2[-1 - 0] - 0[-1 - 8] - 1[0 - 2]$
$= -2 – 0 + 2 = 0$

Property IV

If each element of a row (or a column) of a determinant is multiplied by a constant ‘k’, then its value gets multiplied by k

$$\text{Let } \Delta = \begin{vmatrix} ka_1 & kb_1 & kc_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = k \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Property V

If some or all elements of a row or column of a determinant are expressed as sum of two (or more) terms, then the determinant can be expressed as sum of two (or more) determinants.

$$\begin{vmatrix} a_1+\lambda_1 & a_2+\lambda_2 & a_3+\lambda_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} + \begin{vmatrix} \lambda_1 & \lambda_2 & \lambda_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

Property VI

The value of the determinant remains same if we apply the operation $R_i \longrightarrow R_i + kR_j$ or $C_i \longrightarrow C_i + kC_j$
For example: Using[properties](#) of determinants: Solve

$$\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix} = 0$$

1. $\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix} = \begin{vmatrix} 1 & bc & ab+ac \\ 1 & ca & bc+ab \\ 1 & ab & ac+bc \end{vmatrix}$

$$= \begin{vmatrix} 1 & bc & ab+ac+bc \\ 1 & ca & bc+ab+ca \\ 1 & ab & ac+bc+ab \end{vmatrix} \quad C_3 \longrightarrow C_2 + C_3$$

$= ab+ac+bc \begin{vmatrix} 1 & bc & 1 \end{vmatrix}$

$= (ab + ac + bc) \times 0$
$= 0$

Now try it yourself! Should you still need any help, click here to schedule live online session with e Tutor!

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Reference Links:

- http://en.wikipedia.org/wiki/Determinant
- http://en.wikipedia.org/wiki/Row_and_column_spaces
- http://en.wikipedia.org/wiki/Determinant#Properties_characterizing_the_determinant

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