Solution of system of linear equations using matrix method

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Introduction

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix};\ X = \begin{bmatrix} x_1 \\ x_2 \\ \dots \\ \dots \\ x_n \end{bmatrix};\ B = \begin{bmatrix} b_1 \\ b_2 \\ \dots \\ \dots \\ b_n \end{bmatrix}$$

In this topic, we will discuss the applications of determinants and matrices for solving the system of linear equations in two or three variables and for checking the consistency of the system of linear equations.

Solution of a system of linear equations in two variables

Consider the system of equations,

$$a_1x+b_1y=d_1$$
$$a_2x+b_2y=d_2$$

Let $A = \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix}$ $X = \begin{pmatrix} x \\ y \end{pmatrix}$ $B = \begin{pmatrix} d_1 \\ d_2 \end{pmatrix}$

Then the system of equations can be written as AX=B

$$\begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} d_1 \\ d_2 \end{pmatrix}$$

If A is a non singular matrix then inverse exists and $X=A^{-1}B$

For example: Solve the system of equations $2x+5y=1$

$$3x+2y=7$$

Solution: Here $A = \begin{pmatrix} 2 & 5 \\ 3 & 2 \end{pmatrix}$ $X = \begin{pmatrix} x \\ y \end{pmatrix}$ $B = \begin{pmatrix} 1 \\ 7 \end{pmatrix}$

$|A| = 4 - 15 = -11 \neq 0$, so inverse exists

$$A^{-1} = \frac{1}{-11}\begin{pmatrix} 2 & -5 \\ -3 & 2 \end{pmatrix}$$

$X = A^{-1}B$

$$= \frac{1}{-11}\begin{pmatrix} 2 & -5 \\ -3 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ 7 \end{pmatrix}$$

$$= \frac{1}{\ } \begin{pmatrix} -33 \end{pmatrix}$$

Hence x = 1 and y = -1

Solution of a system of linear equations in three variables

Consider the system of equations $a_1x+b_1y+c_1z=d_1$

$a_2x+b_2y+c_2z=d_2$

$a_3x+b_3y+c_3z=d_3$

As in the previous case, take the coefficients of x, y and z as A unknown values as X and constants after equality sign as B.

$$A = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix} X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

Here also, we can write the system of equations as AX=B

If A is non-singular matrix then inverse exists and $X=A^{-1}B$

Example: The sum of three numbers is 6. If we multiply third number by 3 and add second number to it, we get 11. By adding first and third numbers, we get double of the second number. Represent it algebraically and find the numbers using matrix method.

Solution: Let first, second and third numbers be x, y and z respectively.

According to the given conditions, we have

$x + y + z = 6$

$y + 3z = 11$

$x + z = 2y$

This system can be written as AX = B, where

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{pmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$$

$$|A| = 1(1 + 6) - 2(0 - 3) + 3(0 - 1) = 9 \neq 0$$

Hence inverse exists, so X=A-1B

$$A^{-1} = \frac{1}{9}\begin{pmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{pmatrix}$$

$$X = (1/9) \begin{pmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{pmatrix} \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$$

$$= (1/9) \begin{pmatrix} 42-33+0 \\ 18+0+0 \\ -6+33+0 \end{pmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = (1/9) \begin{bmatrix} 9 \\ 18 \\ 27 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Hence x = 1, y = 2 and z = 3

Consistent System :

A system of equations is said to be consistent if its solution one or more exists.
For example: The system of equations given above, 2x + 5y = 1 and 3x + 2y = 7 are consistent since they have unique solution.

Inconsistent System :

A system of equations is said to be inconsistent if its solution does not exist.

Conditions for the consistency of a given system of equations :

Given a system of equations, take the coefficient matrix as A, matrix with unknown things as X and matrix with the constants after the equality sign in the system of equations as B, so that the system can be written in the form AX = B
Case I: Find |A|, If |A|?0, then inverse exists and system has unique solution which is given by $X=A^{-1}B$
Case II: If |A|=0, then find (Adj A)B
- If (Adj A)B?0, then the solution does not exists and the system of equations is said to be inconsistent.
- If (Adj A)B ;=0, then the system may be either consistent or inconsistent according as the system have either infinitely many solutions or no solution.

For example: Examine the consistency of the following system of equations
i) x+2y=2
2x+3y=3

$$A=\begin{pmatrix}1 & 2\\ 2 & 3\end{pmatrix} \quad X=\begin{bmatrix}x\\ y\end{bmatrix} \quad B=\begin{bmatrix}2\\ 3\end{bmatrix}$$

$$|A| = 3-4=-1\neq 0$$

Since |A|?0, the system is consistent and has unique solution.

ii) x+3y=5

2x+6y=8

$$A = \begin{pmatrix}1 & 3\\ 2 & 6\end{pmatrix} \quad X = \begin{pmatrix}x\\ y\end{pmatrix} \quad B= \begin{bmatrix}5\\ 8\end{bmatrix}$$

$$|A| = 6-6=0$$

$$(Adj\ A)B = \begin{pmatrix}6 & -3\\ -2 & 1\end{pmatrix}\begin{bmatrix}5\\ 8\end{bmatrix}$$

$$= \begin{pmatrix}30+ -24\\ -10+8\end{pmatrix}$$

$$= \begin{bmatrix}6\\ -2\end{bmatrix}$$

? 0

Since (Adj A) B ? 0, the system is inconsistent and has no solution.

Now try it yourself! Should you still need any help,click here to schedule live online session with e Tutor!

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Reference Links:

- http://en.wikipedia.org/wiki/Matrix_%28mathematics%29
- http://en.wikipedia.org/wiki/Determinant
- http://en.wikipedia.org/wiki/System_of_linear_equations#Matrix_equation
- http://en.wikipedia.org/wiki/Invertible_matrix

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