

# SYMMETRIC AND SKEW SYMMETRIC MATRICES

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## Symmetric Matrix

$$\text{Mat}_n = \text{Sym}_n \oplus \text{Skew}_n,$$

A **square matrix**  $A = [a_{ij}]$  is said to be **symmetric** if  $A' = A$ , that is,  $[a_{ij}] = [a_{ji}]$  for all possible values of  $i$  and  $j$

For example:  $A = \begin{pmatrix} 5 & 3 & 2 \\ 3 & 1 & -6 \\ 2 & -6 & 0 \end{pmatrix} \quad A' = \begin{pmatrix} 5 & 3 & 2 \\ 3 & 1 & -6 \\ 2 & -6 & 0 \end{pmatrix}$

Since  $A = A'$ ,  $A$  is symmetric.

## Skew - symmetric Matrix

A square matrix  $A = [a_{ij}]$  is said to be **skew symmetric** if  $A' = -A$ , that is  $a_{ji} = -a_{ij}$  for all possible values of  $i$  and  $j$ .

For example:  $B = \begin{pmatrix} 0 & -1 & 5 \\ 1 & 0 & 4 \\ -5 & -4 & 0 \end{pmatrix} \quad B' = \begin{pmatrix} 0 & 1 & -5 \\ -1 & 0 & -4 \\ 5 & 4 & 0 \end{pmatrix}$

Here  $B = -B'$ , so it is skew symmetric.

## Important Results

1) For any square matrix  $A$  with real number entries,  $A + A'$  is a symmetric matrix and  $A - A'$  is a skew symmetric matrix.

Example: For the matrix  $A = \begin{pmatrix} 1 & 5 \\ 6 & 7 \end{pmatrix}$ , verify that

(i)  $(A + A')$  is symmetric matrix

(ii)  $(A - A')$  is skew symmetric matrix

Solution:

(i)  $A + A' = \begin{pmatrix} 1 & 5 \\ 6 & 7 \end{pmatrix} + \begin{pmatrix} 1 & 6 \\ 5 & 7 \end{pmatrix}$   
 $= \begin{pmatrix} 2 & 11 \\ 11 & 14 \end{pmatrix} \dots (a)$

$(A + A')' = \begin{pmatrix} 2 & 11 \\ 11 & 14 \end{pmatrix} \dots (b)$

From (a) and (b),

$(A + A') = (A + A')'$ , so it is symmetric matrix

$$(ii) A - A' = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \dots (c)$$

$$(A - A')' = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \dots (d)$$

From (c) and (d),

$(A - A') = -(A - A')'$ , so it is skew symmetric matrix.

2) Any square matrix can be expressed as the sum of a symmetric and skew symmetric matrix.

$$A = \begin{pmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{pmatrix}$$

Example: Express

As the sum of symmetric and a skew-symmetric matrices

Solution:

Let  $P = \frac{1}{2} [A + A']$  and  $Q = \frac{1}{2} [A - A']$ . We have to show that  $A = P + Q$

$$A = \begin{pmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{pmatrix} \quad A' = \begin{pmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ 1 & 1 & 2 \end{pmatrix}$$

$$P = \frac{1}{2} \begin{pmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{pmatrix} \quad Q = \frac{1}{2} \begin{pmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{pmatrix}$$

$$P + Q = \frac{1}{2} \begin{pmatrix} 6 & 6 & -2 \\ -4 & -4 & 2 \\ -8 & -10 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{pmatrix}$$

Hence  $P + Q = A$ , so  $A$  is expressed as sum of a symmetric and skew symmetric matrices.

Now try it yourself! Should you still need any help, [click here](#) to schedule live online session with e Tutor!

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## Reference Links:

- <http://www.britannica.com/EBchecked/topic/561660/square-matrix>

- [http://en.wikipedia.org/wiki/Symmetric\\_matrix](http://en.wikipedia.org/wiki/Symmetric_matrix)
- [http://en.wikipedia.org/wiki/Skew-symmetric\\_matrix](http://en.wikipedia.org/wiki/Skew-symmetric_matrix)

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