SYMMETRIC AND SKEW SYMMETRIC MATRICES

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Symmetric Matrix

$$\mathrm{Mat}_n = \mathrm{Sym}_n \oplus \mathrm{Skew}_n,$$

A square matrix A = [a_{ij}] is said to be symmetric if A' = A, that is, [a_{ij}] = [a_{ji}] for all possible values of i and j

For example: $A = \begin{pmatrix} 5 & 3 & 2 \\ 3 & 1 & -6 \\ 2 & -6 & 0 \end{pmatrix}$ $A' = \begin{pmatrix} 5 & 3 & 2 \\ 3 & 1 & -6 \\ 2 & -6 & 0 \end{pmatrix}$

Since A = A', A is symmetric.

Skew - symmetric Matrix

A square matrix A = [a_{ij}] is said to be skew symmetric if A' = -A, that is $a_{ji} = -a_{ij}$ for all possible values of i and j.

For example: $B = \begin{pmatrix} 0 & -1 & 5 \\ 1 & 0 & 4 \\ -5 & -4 & 0 \end{pmatrix}$ $B' = \begin{pmatrix} 0 & 1 & -5 \\ -1 & 0 & -4 \\ 5 & 4 & 0 \end{pmatrix}$

Here B = B', so it is skew symmetric.

Important Results

1) For any square matrix A with real number entries, A + A' is a symmetric matrix and A - A' is a skew symmetric matrix.

Example: For the matrix $A = \begin{pmatrix} 1 & 5 \\ 6 & 7 \end{pmatrix}$, verify that

(i) (A + A') is symmetric matrix

(ii) (A - A') is skew symmetric matrix

Solution:

(i) $A + A' = \begin{pmatrix} 1 & 5 \\ 6 & 7 \end{pmatrix} + \begin{pmatrix} 1 & 6 \\ 5 & 7 \end{pmatrix}$

$= \begin{pmatrix} 2 & 11 \\ 11 & 14 \end{pmatrix}$ ... (a)

$(A + A')' = \begin{pmatrix} 2 & 11 \\ 11 & 14 \end{pmatrix}$ ... (b)

From (a) and (b),

(A + A') = (A + A')', so it is symmetric matrix

(ii) $A - A' = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ … (c)

$(A - A')' = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ .. (d)

From (c) and (d),
(A - A’) = -(A - A’)’, so it is skew symmetric matrix.

2) Any square matrix can be expressed as the sum of a symmetric and skew symmetric matrix.

Example: Express $A = \begin{pmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{pmatrix}$

As the sum of symmetric and a skew- symmetric matrices
Solution:
Let P = ½ [A + A’] and Q = ½ [A - A’]. We have to show that A = P + Q

$A = \begin{pmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{pmatrix}$ $A' = \begin{pmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{pmatrix}$

$P = ½ \begin{pmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{pmatrix}$ $Q = ½ \begin{pmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{pmatrix}$

$P + Q = ½ \begin{pmatrix} 6 & 6 & -2 \\ -4 & -4 & 2 \\ -8 & -10 & 4 \end{pmatrix}$

$= \begin{pmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{pmatrix}$

Hence P + Q = A, so A is expressed as sum of a symmetric and skew symmetric matrices.

Now try it yourself! Should you still need any help, click here to schedule live online session with e Tutor!

About eAge Tutoring:

Reference Links:

- http://www.britannica.com/EBchecked/topic/561660/square-matrix

- http://en.wikipedia.org/wiki/Symmetric_matrix
- http://en.wikipedia.org/wiki/Skew-symmetric_matrix

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